

# A real world application of secure multi-party computations

Duplicate bridge for cheapskates

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# Outline

Example deal

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Protocol

Flaws and corrections

Case study

Error detection/correction

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# Example deal

## 1st permutation

Order the suits: Clubs Hearts Spades Diamonds

1143 2323 4422 1143 2411 4143

1332 4344 1223 2433 1211 3242

4224

## 2nd permutation

3231 1224 1243 4421 1233 4421

1311 1432 3332 2441 2244 3332

4141

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♠ 82  
♥ A3  
♦ AQ985  
♣ Q854

♠ KT95  
♥ KJ9  
♦ 432  
♣ KJ6

♠ A43  
♥ T86  
♦ J76  
♣ 9732

♠ QJ76  
♥ Q7542  
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# Multi-party protocols

## Traditionally

- ▶ Secret inputs to each party generating a shared result
- ▶ Computations done on computer

## For duplimating

- ▶ Secret result, known inputs
- ▶ 'Computations' done by humans
- ▶ Intermediate state can be secret

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# Attacker model

- ▶ Assume the players are inherently trustworthy
  - ▶ They can cheat anyway if not
  - ▶ Most players are trustworthy
- ▶ Players are sufficiently intelligent to make use of small amounts of information
- ▶ Main security goals:
  - ▶ Ensure neither dealer can deduce much about the hands while dealing...
  - ▶ ...and having seen one of the hands.

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# Protocol specifics

1. Generate random  $P_T$ ;  $T = \{S\}_{E_{P_T}}$
2. Discard  $P_T$
3. Generate random  $P_1$  and  $P_I$
4. Calculate  $P_2$  s.t.  $T = \{\{S_{P_I}\}_{E_{P_1}}\}_{E_{P_2}}$
5. Give  $P_I$  &  $P_1$  to dealer 1
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# Flaws and corrections I

## Suit of the first card dealt

- ▶ Last thirteen cards in  $P_1$  same suit.
- ▶ Likely that there will be a 1 in the last 13 numbers of  $P_1$ .
- ▶ Implies first card of  $P_2$  is that suit.
- ▶ First hand dealt in  $P_2$  does not have a void in that suit.

## Solution

Randomize the order of the suits in  $P_1$ .

## But...

Hands must be shuffled before going into the boards, else the second dealer can infer the suit order from the order of the cards in their hands.

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# Flaws and corrections II

## Locating high cards

- ▶ High cards from first suit will be at the bottom of some of the piles
- ▶ One of positions  $\{13, 26, 39, 52\}$  in  $P_2$  will hold an ace.

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Randomize the number of cards in each pile at the end of  $P_1$ .

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## Case study

- ▶ Two trials, 3 sessions in November–December 2007, 6 sessions in January–March 2008.
- ▶ Approximately six dealers in total, three pairs.
- ▶ Time to deal 28 boards consistently 10–15 minutes.
- ▶ Observed error rate 4–6 boards, with one perfect result.

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## No detection

- ▶ Ignore errors.

## Detection only

- ▶ Check at some point during the play against the hand record for that board.

## Detection and correction

- ▶ Check the first time the board is played using curtain cards.
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## Trial error results

Session	Failures	Recoverable Errors
14/03/08	1	3
07/03/08	2	3
22/02/08	0	0
15/02/08	2	2
07/02/08	3	4
31/02/08	4	2
30/11/07	5	2
16/11/07	7	1
01/11/07	4	1

Table: Errors in each session